Prospective teacher's expectation of students' critical thinking process in solving mathematical problems based on Facione stages

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M Tohir[1*], M Maswar[1], M Mukhlis[2], W Sardjono[3], and E Selviyanti[4]

[1]Universitas Ibrahimy, Situbondo, Indonesia
[2]Institut Agama Islam Negeri Jember, Indonesia
[3]Bina Nusantara University, Indonesia
[4]Politeknik Negeri Jember, Indonesia

E-mail:matematohir@ibrahimy.ac.id

***Abstract*:** The professional teachers have to know about the students' critical thinking process. So, a teacher can teach mathematics kindly to students at school. Then, all prospective teachers have to like that to be ready to be a good facilitator in the process of learning mathematics at school and others. This study aimed to describe the critical thinking process expected by prospective teachers to students in solving mathematical problems based on critical thinking stages of Facione's theory. The research method used is descriptive research employing a qualitative approach. The subject of the study was the prospective mathematics teachers who were students of the Mathematics Study Program at Ibrahimy University, Situbondo, Indonesia. The data were collected by giving an essay test item to prospective teachers. The test was given to collect data related to the critical thinking process of prospective mathematics teachers. Moreover, the data collected through observations, tests, and interviews were analyzed and tested for their validity by using triangulation. The data analysis technique used is qualitative descriptive data analysis. The results of this study indicate that: (1) each of group subjects' expectations tend to have different expectations for students' critical thinking processes; (2) the high-group subject tend to expect that students follow and complete all of the critical thinking process stages according to Facione's theory; (3) the moderate and low-group subjects tend not to expect their students to follow and complete all of the critical thinking process stages according to Facione's theory; and (4) the percentage of indicators achievement of critical thinking processes expected by prospective teachers to students in solving problems based on Facione theory was revealed to be; Identify (57.14%), Define (50%), Enumerate (78.57%), Analyze (50%), List (39.29%), and Self-Correct (32.14%). The results of this study can serve as the fundamental to be discussed within the teacher's summit in finding out the critical thinking process expected by teachers towards the students, because it will have a positive impact both on themselves and their students.

1. Introduction

Critical thinking is a cognitive skill that is embedded in a person logically and effectively to gather information to be interpreted, analyzed, and evaluated to get a particular valid decision. Critical thinking is needed to check the accuracy of information so that it can decide whether the information is worth rejecting or accepting [1]. Critical thinking is also one of the 21st-century skills in which it refers to the learning process that requires students to have competence in critical and creative thinking, communicative, and collaborative [2] [4] [5]. Critical thinking skills must be acquired by

everyone so that they are always careful in acting and making decisions in regards to the information obtained, because the information itself might not be accurate. The 21st-century skills are very important to be noticed, learned, and understood by teachers and lecturers, especially students as prospective teachers.

Critical thinking skills of students need to be sharpened and improved gradually in a continuous manner for all levels of education, because students who can think critically will be able to solve problems effectively [3]. Someone who has the ability to think critically will be very influential in their daily lives, who will always make the right and good decisions. This is strengthened by Ennis who states that critical thinking is a logical and reflective thinking that is focused on making decisions about what to believe or what to do [2]. Therefore, the role of prospective teachers is very important in learning various learning models that are innovative and creative, so that the critical thinking process of students can always be sharpened gradually and continuously in every moment; to make the critical thinking process itself always develops. According to Jufri, he explained that critical thinkers always go through several stages in their actions consisting of formulating problems, giving arguments, making deductions, inducing, conducting evaluations, then making decisions and determining actions [6]. This particular stage has similar characteristics with the critical thinking process stages formulated by Facione in which he explains that to find out the mental activity of students in critical thinking in solving a problem, can use the stages of the *Identify, Define, Enumerate, Analyze, List,* and *Self-Correct* [2].

The results of a study conducted by Cherubini show that the critical thinking of prospective teachers becomes more concentrated as the investigation progresses from case one to case four. However, the critical analysis of teacher candidates most of the time is a reflection of memories and prejudices from their own experiences during their elementary and middle school period [7]. The results of research conducted by Kurniawan on critical thinking skills show that students' mathematical critical thinking skills that are subjected to problem-solving learning are better than students' mathematical critical thinking skills that are subjected to traditional learning [8]. Likewise, the results of research conducted by As'ari, Mahmudi, & Nuerlaelah also show that the majority of the critical thinking dispositions of mathematics teachers are at the level of non-critical thinkers. Only a few of them to be considered as the critical thinkers who emerge, and very rarely at the level of developing critical thinkers. Hence, it can be said that mathematics teacher candidates have not thought critically [9].

Based on the results of these three studies mentioned, the further research for prospective mathematics teachers about their thought processes is still essential to be done continuously, especially on how critical thinking processes are expected from their prospective students in solving the mathematical problems. The strength point of this study is that prospective mathematics teachers were asked to describe their critical thinking processes when solving mathematical problems seen as the expectations for their students' critical thinking processes. Hence the purpose of the research conducted by the author is to describe the critical thinking process of prospective mathematics teachers in solving mathematical problems based on the stages of the critical thinking process formulated by Facione, the critical thinking process expected by prospective teachers serves as the expectations for their students in solving mathematical problems.

2. Research Methods

2.1 Research design

The research method used in this study was a triangulation method by using qualitative and quantitative approaches. Qualitative research has the characteristics of having a natural background (context of a wholeness), humans as a tool or instrument, using qualitative methods, applying inductive data analysis, having data based theory preparation, using descriptive data, focusing more in the aspect of the process than results, having the existence limits determined by focus, the existence of specific criteria for the validity of the data, the design is temporary, and the results of the study are the result of joint decisions [2] [3] [10]. Descriptive research with a qualitative approach is research that uses qualitative data and then described to produce a clear and detailed depiction of the critical thinking process of prospective teachers in solving mathematical problems based on the stages of

Facion's critical thinking process. According to Saryono, qualitative research is a research that is used to investigate, describe, explain, discover the quality or features of social influence that cannot be explained, measured, or illustrated through a quantitative approach [2].

2.2 Participants

The subjects of this study were 25 students of the Mathematics Study Program at Ibrahimy University, Situbondo. Students as prospective teachers were expected to describe the process of critical thinking in solving the problems focusing on the thought process itself as the expectation for their students. There are six stages used as the indicators of critical thinking processes in solving problems is shown in Table 1 below.

Table 1. Indicator of critical thinking process based on facione's stages.

Critical Thinking Process Stages	**Indicator of the Critical Thinking Process**
Identify (I)	Subjects were able to mention the main idea of the problem at hand. Subjects were able to communicate back the main idea of the problem in their own words orally, in writing, pictures, or diagrams.
Define (D)	Subjects were able to state what is known and what is asked in the problem. Subjects were able to inform what is not used or not needed to solve the problem.
Enumerate (E)	Subjects were able to mention strategic choices in solving problems. Subjects were able to find the right and sensible strategy in solving problems.
Analyze (A)	Subjects were able to analyze strategy options to select completion procedures. Subjects were able to predict the best answer based on the selected completion procedure.
List (L)	Subjects were able to state the exact reasons for the chosen completion procedure. Subjects were able to state the reasons that the answer obtained is the best answer.
Self-Correct (S)	Subjects were able to thoroughly recheck the completion procedure, to obtain the best answer. Subjects were able to draw valid conclusions that the answer obtained is the best.

Source: Munawwarah, Laili, & Tohir [2].

2.3 Data collection

Data collection techniques are the most important Stage in research because they determine the success of getting accurate data, which then analyzed based on the students' answers to the given problem. The data were collected by giving a test. The test was given to collect data related to the critical thinking process of prospective mathematics teachers. The data collected was reduced, presented, concluded, and verified. Data verification is done by the triangulation method, peer checking, and extended observation. The triangulation is an attempt to check the accuracy of data or information obtained by researchers from different points of view by reducing the bias as much as possible in the process of collecting and analyzing the data [3] [11].

2.4 Data analysis

Presentation of data in qualitative research can be done in the form of brief descriptions, charts, associations between categories, flowcharts, and any other similar techniques. Miles and Huberman as quoted by Sugiyono suggested that the most frequent form of data display for qualitative research data in the past is narrative text [12]. The analysis was done after the process of giving a written test, and then the results of the written test were used as a reference in constructing interview guidelines. Data analysis was performed to uncover the critical thinking process of prospective mathematics teachers in solving mathematical problems. The process of data analysis in this study was based on the process of qualitative data analysis, which consists of the process of reducing data, presenting data, and drawing conclusions. Presentation of data includes the classification and identification of data, stating organized and categorized data sets so that a conclusion can be drawn [3] [5]. The conclusions obtained will be additional and supporting material in conducting further research.

3. Research Results

Based on the results of the student's GPA (Grade Point Average) score as prospective teachers, they were divided into three categories namely; the high, moderate, and low-group subjects. The second data was obtained based on the test results of critical thinking processes in solving mathematical problems. The critical thinking processes of prospective teachers in solving mathematical problems itself were in the form of an essay test item. Then, the researcher conducted an interview with the students to clarify the basics of their critical thinking process that had been written on the answer sheet in solving mathematical problems based on the critical thinking process indicators of Facione's stages. The results of prospective teachers GPA scores for the high, moderate and low group categories can be described in the following Table 2.

Table 2. The results of prospective teachers GPA scores.

Categories	Minimum	Maximum	Mean		Std. Deviation	Variance	Kurtosis	
	Statistic	Statistic	Statistic	Std. Error	Statistic	Statistic	Statistic	Std. Error
High	92	95	93.75	0.412	1.165	1.357	-1.613	1.481
Moderate	89	92	90.56	0.294	0.882	0.778	0.144	1.400
Low	76	89	85.50	1.592	4.504	20.286	2.391	1.481

3.1 Critical thinking process in the identify (I) stage

The critical thinking process expected by the high-group subjects in the stage of Identify (I) begins with reading the problem carefully and repeating the process to have a good understanding about the problem when the content is not yet understood; being capable of mentioning the main idea in the questions itself by imagining that the Hall discussed within the question is taken a shape of a block or illustrating it in the form of a drawing so that it is easily understood; being capable of understanding which areas (the Hall's walls) should be painted; being able to communicate the main idea yet again in written form; being able to analyze the building of the Hall in relation to the painted walls of the Hall, questioning whether there are no doors or windows on the wall, then also in case there is a difference in the aspect of the paint's brand so that it might give a difference value for the price of the paint too; and being capable of formulating assumptions so that the problem can be solved with an existing concept, for the information contained within the test item is incomplete so that the question arises whether there is no door or window in the hall.

The critical thinking process expected by the moderate-group subjects at the Identify (I) stage starts with reading the problem and underlining the important information on the problem itself. However, they did not understand the information contained in the problem. The subject paused while scratching his head trying to remember what he had learned before and then trying to re-read the problem several times until he understood. Only a small number of subjects who were able to make assumptions so that the problem can be solved with mathematical concepts that had been studied before.

The critical thinking process expected by the low-group subjects in the Identify (I) stage begins with reading the problem and being able to explain the problem verbally about the information contained. However, most of the subjects did not write down the answer on the given sheet about their expectations, they gave an incorrect answer in formulating the main problem related to the material used in the problem, and were unclear in explaining the meaning of the problem.

3.2 Critical thinking process in define (D) stage

The critical thinking process expected by the high-group subjects in the Define (D) stage begins by writing down the part that is known and asked within the problem both verbally and in written form. The subject was able to provide an explanation of the process in writing down what is known and asked as well as the arrangements that had been prepared. The subject was able to thoroughly state what information could be used and what information was not used in solving the problem by paying attention to the area of the wall to be painted and completing the length, width, and height in the illustration of the drawing that had been made previously. The subject explained the exact reasons for the assumptions made in the problem effortlessly, both in spoken and written form so that the problem could be solved in a mathematical concept that had been studied previously.

The critical thinking process that is expected by the moderate-group subject at the Define stage (D) begins by writing down what is known and asked verbally or in written form on the answer sheet. Most subjects were able to provide an explanation of the process in writing down what is known and asked, but only a small number of subjects who were able to explain the assumptions that had been arranged. The subject was able to mention what information could be used and what information could not be used in solving the problem.

The critical thinking process expected by the low-group subjects in the Define stage (D) begins by explaining the problem verbally about what is known and asked. Only a small number of subjects could explain what is known and asked in their own language. Meanwhile most subjects did not write down the answer on the answer sheet, and the subject was not able to provide an explanation of the determination about what is known and asked.

3.3 Critical thinking process in the enumerate (E) stage

The critical thinking process expected by the high-group subjects in the Enumerate (E) stage begins by stating the choice of strategies and finding appropriate and sensible strategies in solving the problem based on self-composed assumptions and the prior knowledge experienced by the subjects. The subject was able to prepare the second strategy when the first strategy failed and it could also be used as a checking process on the results obtained. The subjects were able to find the connection between the information known to the problem related to the information so that they could easily find the right strategy based on the assumptions that had been prepared.

The critical thinking process expected by the moderate-group subjects in the Enumerate (E) stage starts with making a plan about the strategy and choosing the right and sensible strategy to solve the problem based on self-composed assumptions and prior knowledge, and most subjects thought of other strategies which are more effective and valid, while a small number of subjects did not think of other strategies because they believed that the answer was correct. Most subjects were able to provide reasons and strong evidence for the chosen strategy. Moreover, most subjects were also able to find a connection between the known information and the one which is related to the problem.

The critical thinking process expected by the low-group subjects in the Enumerate (E) stage begins by planning any strategies that can be used to solve the problem by illustrating the Hall's image into a block so that it can be identified which parts of the wall are said to be painted to use the formula of the surface area of the block itself without having a base or lid. However, they did not have another different strategy that can be used to solve the problem when the strategy thought could not solve the problem.

3.4 Critical thinking process in analyze (A) stage

The critical thinking process expected by the high-group subjects in the Analyze (A) stage begins with analyzing the choice of strategies used and explaining on how to use the chosen completion procedure.

The subject was able to analyze the reserve choice strategy (second strategy) if it is necessary, and using an advanced strategy to be used in checking the validity of the answers obtained. The subject was able to guess the best answer based on the chosen completion procedure with the best-selected strategy.

The critical thinking process expected by the moderate-group subject in the Analyze (A) stage starts with analyzing the strategic plan and the selected strategy in solving the problem based on self-composed assumptions and prior knowledge. Most subjects could analyze other strategy choices that are more effective and accurate as a backup strategy. Most subjects were able to analyze the reasons and gave evidence for the chosen strategy. Most of the subjects were able to analyze and find the connection between the information known with the information related to the problem so that they were able to assume that their answer was the correct one.

The critical thinking process expected by the low-group subjects in the Analyze (A) stage starts with analyzing the choice of strategies used. However, they could not explain how to use the appropriate completion procedure correctly so that the answers to most subjects were incorrect.

3.5 Critical thinking process in list (L) stage

The critical thinking process expected by the high-group subjects in the List (L) stage begins by giving an appropriate reasons why using such completion procedure, that are; making an image illustration first, after that looking for the total area of the block, then after finding the surface area of the hall, students are expected to look for the cost of The painted area within the inner walls. In this case, the students are not only expected to multiply the area of the Hall with the cost of paint/m2, but also to look for the selected area. This is because the main problem of the test item is to find out the cost of the painting process within the inner walls, not the entire surface area of the Hall. Then it must be reduced first with the cost of the roof and the floor so that the cost of the intended paint costs is Rp. 5.600.000,-.

The critical thinking process expected by the moderate-group subjects in the List (L) stage begins by giving reasons why using such completion procedure, but the subjects were unable to give reasons for the answers obtained as the best answer, even they were scratching their head which indicates that they were confused to express the reason. After a few minutes, the subjects finally could reveal the reason in which they claimed that the procedure used was based on their previous experience and explained that such problems, most of the time, had been discussed when they were in middle school, that is using the surface area of the block without the base and lid: 2 (length × height + width × height).

The critical thinking process that is expected by the low-group subjects in the List (L) stage begins by giving a reason why using such completion procedure, but the images illustration made were not appropriate to depict the Hall wall to be painted so that some of the subjects' answers were incorrect. Most subjects assumed that the information contained in the test item is complete and comprehensive so that they thought that it was unnecessary to make other assumptions, when in fact it was necessary to arrange particular assumption so that the problem can be solved mathematically.

3.6 Critical thinking process in the self-correct (S) stage

The critical thinking process expected by the high-group subjects in the Self-Correct (S) stage begins by re-examining the results of their work as a whole towards the completion procedures that had been constructed by carefully examining and recalculating the arithmetic operations implemented from the beginning up to the last part, considering whether there was missed or there was an error in the calculation made to draw an accurate conclusion. There were few of the subjects that re-examine using another strategy (the second selected-strategy) to conclude that the answer is correct so that they got the best results.

The critical thinking process expected by the moderate-group subject in the Self-Correct (S) stage begins by re-examining the problem-solving results by trying to re-do the procedure so that they believe that the answer obtained was the correct one, because they assumed that the method used was the same method that had been studied before so that when re-checking the answer, it remained the

same. As a result, they believed that the answer was correct. However, most subjects did not try to use other strategies to check the validity of the answers obtained.

The process of critical thinking expected by the low-group subjects in the Self-Correct (S) stage begins by recalculating the arithmetic operations from the first up to the last part of the solution as it could be seen by the presence of a smear on the answer sheet. However, only a small number of subjects got the correct answer.

3.7 Results of the test for critical thinking process indicator's achievement

Data related to the results of the indicators for critical thinking processes expected by prospective teachers based on the six stages of critical thinking formulated by Facione can be seen in Figure 1.

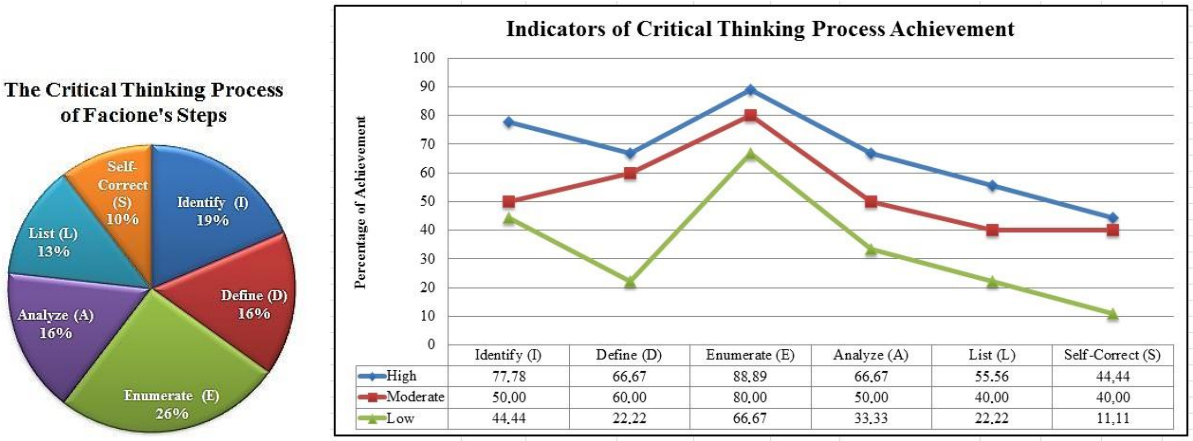
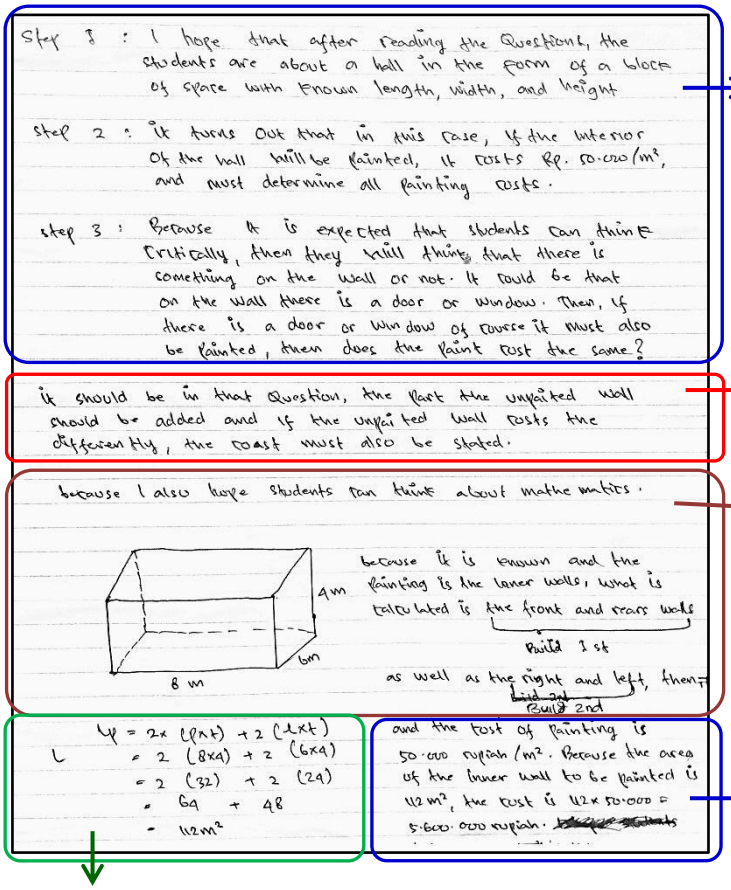


Figure 1. Percentage of critical thinking indicator's achievement of prospective teacher.

According to the average score of the percentage for critical thinking process indicator's achievement in Figure 1, it reveals that there are very significant differences between the high, moderate, and low-group subjects' average score for the critical thinking achievement's percentage of prospective teachers. The achievement results in the Identify (I) stage were found to be 77.78%; 50%; and 44.44%. Hence the difference between the achievements is 27.78% and 5.56%. Meanwhile, the achievement results in the Define stage (D) were 66.67%; 60%; and 22.22%. In other words, the difference between the achievements score obtained is 6.67% and 37.78%. The results of achievement in the Enumerate (E) stage were revealed to be 88.89%; 80%; and 66.67%. In this particular case, the difference within the achievement score is 8.89% and 13.33%. The results achieved in the Analyze (A) stage were 66.67%; 50%; and 33.33%. Thus, the difference between the achievement score obtained equal to 16.67. Furthermore, the achievement results in the stage of List (L) were found to be 55.56%; 40%; and 22.22% respectively. Hence that the difference in the achievement score is 15.56% and 17.78%. Likewise, the achievement results in the Self-Correct (S) stage were 44.44%; 40%; and 11.11%. In this matter, the difference within the achievement score is 4.44% and 28.89%. Meanwhile, the percentage of the achievement result of the critical thinking process as a whole was recorded; the Identify (I) stage was 57.14% (19%); the Define stage (D) was 50% (16%); the Enumerate stage was (E) 78.57% (26%); the Analyze (A) stage was 50% (16%); the List (L) stage was 39.29% (13%), and the Self-Correct (S) stage was 32.14% (10%). Therefore, the largest and smallest percentage of the achievement is found to be in the stage of Enumerate (E) and Self-Correct (S).

4. Discussion

The following Figure 2 reveals an example of the results of the subject's answers as the expectations of students' critical thinking processes in solving mathematical problems.



Identify (I)
The initial description of the subject's expectations of students' critical thinking processes in solving problems.

The subject criticizes the content of the problem in which it lacks some information so that an assumption must be made to make it works mathematically.

Define (D) & Enumerate (E)
Writing down what is known while choosing the strategy to be used.

Self-Correct (S)
Drawing a valid conclusion that the answer obtained is the best one.

Analyze (A) & List (L)
Analyzing and applying the chosen completion procedure to get the correct answer

Figure 2. Results of subject A's answer.

Based on the results of Subject A's answers and the analysis in Figure 2, it seems that it needs to be investigated further for the answers appear to be very interesting by conducting a distinctive interview to the subject, especially on how the subject's expectation about the critical thinking process to the students in solving mathematical problems. The following are the results of the interview between the Researcher (R) and Subject (S).

R : What exactly do you expect from your students to face this kind of problem?
S : I expect that the student will be able to understand the questions given by reading the questions first, then paying attention to the information enclosed in the questions about a Hall that its length, width, and height has been known, and whether the information complete or not so that it can be solved mathematically.
R : Do you expect the students to re-communicate the main ideas contained in the problem, either to communicate it in their own words orally or in writing from or drawings or diagrams?
S : Yes sir, I hope that students can re-communicate the main ideas contained in the problem, although each student has their own way of thinking, which of course is different, in understanding the problem, and I hope students can make illustrations of the Hall image in the form of blocks to communicate the main ideas in this problem more easily.
R : Why do you expect the students to choose such picture illustrations?

S : Because I would like to know whether the student can think critically or not about the problem that the costs of paint needed are different from an overall Hall's walls covered entirely, and those with the walls having doors and windows.
R : Why did you choose and use such strategies to get the answers?
S : I want the student to think broadly about the questions to be worked on. Because in this particular matter, it was only mentioned that the painting was on the walls of the Hall, in which it means that the floor and roof were impossible to paint. If the Hall wall has windows, the window will not be painted with the same type of paint as the wall paint. If the window is made of glass, you don't need to paint it, sir.
R : How do you know that the answer you got is correct?
S : That's because it is just my expectation for students in solving the problem. So the answer is based on my logic, sir. I expect that the students can make corrections to the answers obtained by taking looking at the mathematical operations based on the completion procedure used to get the correct answer.
R : How can you prove that your answer is correct?
S : Yes, I have proved it at that time, sir. I worked out the problem by imagining the classroom used as the Hall, in which I looked for the overall surface area of the Hall first, then reduced the area of the floor and roof so that the cost used was the result of the reduction by assuming that the Hall's walls were covered by nothing, because if the walls of the Hall had doors and windows, then the total cost would be different.

Based on the results of the answers, analysis, and interviews of the high-group subjects, it showed that the expectation about the critical thinking process for students is that they are able to meet all the indicators of the critical thinking process based on the stages proposed by Facione well. The subject can criticize and analyze the problems given. The results of this study strengthen the research results obtained by Fikri, Mardiyana, & Kuswardi showing that subjects with high interest in learning mathematics can solve mathematical problems to the stage of researching the process and correcting as necessary, properly, and perfectly, as well as subjects with high learning interest have the ability of doing interpretation, analysis, evaluation, conclusions, explanations, and self-regulation in critical thinking [13]. The results of this study are also in line with the statement expressed by Facione saying that someone will be able to think critically well if they meet the criteria of interpretation, analysis, evaluation, inference, explanation, and self-regulation [14]. This is also confirmed by Hasan saying that students' thinking processes will run properly as expected when the theoretical information processing components existing from stimulus to long-term memory in students function properly and correctly too [15].

In the Identify (I) stage, the subject can read the problem clearly, mention the main ideas in the problem, and re-communicate the main idea by writing or drawing. This is in accordance with the statement proposed by Facione who said that in the interpretation, the subject can express the message or meaning of the data [14]. In the Define (D) stage, the subject has revealed what is known and asked about the problem. The subject can thoroughly state what information can be used and what information is not used in solving the problem. This is in accordance with the opinion of Facione saying that in the inference criteria, the subject is able to infer and consider information from the data, and for explaining the criteria, the subject is able to convince the results of the reasoning methodologically [14]. According to the results of the study conducted by Wulantina, she concluded that students with high ability in the preparation stage, are able to identify problems that are asked well, they also can choose the information needed and those which are not in solving problems correctly [5]. In the Enumerate (E) and Analyze (A) stages, the subject is able to name the strategy choices and find the right and sensible strategy to solve the problem. The subject is able to analyze the selected strategy used and explains how to use the chosen completion procedure appropriately. This is confirmed by Facione saying that in the criteria of analysis the subject is able to identify the intent and relationship of the concept to express reasons or opinions [14]. In the List (L) stage, the subject is able to give reasons why in the completion procedure itself, such a method is used. This is also in accordance with the opinion of Facione who said that on the criteria evaluation, the subject is able to

assess the credibility of the statement regarding his opinion [14]. On the other hand based on a study conducted by Saefudin, he concluded that when applying ideas, students with high mathematical abilities do not make mistakes in problem-solving, and feel challenged to solve problems in a variety of ways and answers [16]. In the Self-Correct (S) stage, the subject is able to re-examine the results of his work as a whole towards the completion procedure, then re-examine it again carefully whether anything is missing or an error in the calculation to draw valid as well as accurate conclusions. This is in accordance with the opinion of Facione that on the self-regulation, the subject can monitor, assess, and correct the results of his work [14].

The results of this study strengthen the results of a research conducted by Afandi showing that "subjects with high mathematical ability, in the Identify stage, can determine the subject matter. In the Define stage, the subject can define the facts according to the problem. In the Enumerate and Analyze stage, the subject lists reasonable choices of answers and analyzes them. In the List stage, the subject gives a reason to solve the problem. In the Self-Correct stage, the subject checks the overall results of his work" [17]. Therefore, students who have high mathematical abilities could be seen on how they process their critical thinking in dealing with problems, in which they would dig some information on the problem carefully and thoroughly so that they will question whether the information is insufficient or not, then compose their own assumptions so that the problem can be solved by mathematical completion procedures to get valid results after re-checking the answers obtained, if it is necessary, they try to use other strategies that are more efficient and practical.

According to the results of the answers, analysis, and interviews of the moderate-group subjects, it revealed that there were two or three stages that were forgotten to be written on the answer sheet. This was made clear by the results of the interview that the subject was less able to give the proper reasoning for the chosen completion procedure and was unable to make any correction appropriately within the List (L) and Self-Correct (S) stages. The results of this study underline and strengthen the results of a research conducted by Amir showing that in the stage of List (A), the subject did not give the appropriate reasoning for the methods and answers used [6]. The results of this study also strengthen the results of research conducted by Khusnawiyati & Teguh Budiarto which shows that the subjects having moderate mathematical ability were not able to go through several criteria of the critical thinking process [18]. The results of this study fortify the results of a research conducted by Fikri et al. which shows that subjects having a moderate interest in learning mathematics are able to solve problems up to the stage of assessing the situation and making initial decisions imperfectly and they have the ability of doing interpretation, analysis, and evaluation, as well as having the ability to draw conclusions, but they are lacking in giving a perfect explanations in performing critical thinking process [13].

Based on the results of the answers, analysis, and interviews of the low-group subjects, it suggests that their expectation for students' critical thinking process is that they are not able to meet all the indicators of critical thinking processes based on the stages of Facione. This was underlined by looking at the results of the interview that the subjects were not able to analyze, most of them did not write what was known and asked, they could not provide the appropriate reasons for the chosen completion procedure, and they were unable to re-check the results of the answer at the Analyze (A), Define (D), List (L), and Self-Correct (S) stage. In other words, the low-group subjects tend to be less able to do problem-solving stages based on the critical thinking process stages according to Facione's theory. The results of this study strengthen the results of previous studies conducted by Fikri et al. which shows that subjects with low interest in learning mathematics cannot solve linear program problems according to Facione's problem-solving stages. This is due to the fact that the low-group subjects rarely remember or even they forget about all of the information that has been obtained previously for they rarely repeat, relearn, and train themselves by doing similar questions at their home [13]. This is in line with Soekamto who said that a person can forget information that has been obtained because he fails to change short-term memories into long-term memories due to the lack of repetition or because he cannot group or put together the information he gets [5].

Therefore, it can be concluded that not all subjects can perform the six stages of the critical thinking process according to Facione's theory perfectly. It seems that they need some more time to think, in which at a time like that then cognitive conflict will occur. This is in accordance with the

opinion proposed by Machmud who says that solving mathematical problems is the ability of students to be able to understand the problem through identifying the elements that are known, asked and the adequacy of the elements needed, making/compiling a strategy and representing it, choosing/implementing strategies in the completion procedure to get a solution, and checking the accuracy of the solution and reflecting it [19]. According to Gartmann & Freiberg, they said that in doing problem-solving there is a process of realizing and organizing the thought about how students make approaches to the problems, choosing strategies used to find solutions and asking themselves about the problem [16].

5. Conclusion

Based on the research results, analysis, and discussion in this study, it can be concluded that: (1) all the research subjects expect students to sharpen and improve their critical thinking processes gradually and continuously based on their direction and guidance in solving mathematical problems, so that when discussing a certain material in the class, they can easily accept and understand the material properly; (2) the expectations of the subjects within different group categories tend to have different critical thinking processes which are reflected in their the characteristics and the prior knowledge of them. This is shown by the results of the analysis in this study, which suggests that the subjects who are in the category of the high groups, on average, are able to explain what is written/done based on the stages of critical thinking proposed by Facione. The subjects in the moderate-group category are able to provide an explanation of what was written/done, yet it was incomplete based on the critical thinking stages proposed by Facione. Whereas subjects in the low-group category were, on average, unable to provide an explanation of what was written/done based on the critical thinking stages proposed by Facione; and (3) the critical thinking process expected by the high-group subjects towards students in solving problems through the six stages according to Facione's theory; Identify (I), Define (D), Enumerate (E), Analyze (A), List (L), and Self-Correct (S). In the Identify stage (I), the subjects describe their expectations based on the information contained in the problem by making some connection to the other related information, in the Define stage (D), the subjects write down what is known and asked, in the Enumerate (E) and Analyze (A) stages the subjects choose and use a well-chosen strategy, in Stage List (L) stage, the subjects give a reason for the completion procedure used, and in the Self-Correct (S) stage the subjects verify the mathematical operation use within the completion procedure to make sure that the answer obtained is correct, and some other subjects are able to prove it by using other strategies to ensure that the answer is the same and correct. Moreover, the critical thinking process expected by the moderate-group subjects to students in solving problems is that they are only able to go through the four stages according to the Facione theory, which are Identify (I), Define (D), Enumerate (E), and Analyze (A) stages. While the critical thinking process expected by the low-group subjects to students in solving problems is that they only able to go through two stages of the Facione theory; Identify (I), and Enumerate (E) stages.

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